# ECONOMICS #1: CALCULATION OF GDP

Dr. Paul Kutasovic
Mathematics by Dr. Sharon Petrushka

## Introduction

Gross Domestic Product or GDP is the broadest measure of the health of the US economy. Real GDP is defined as the output of goods and services produced by labor and property located in the US. Real GDP is an important indicator to track because it provides the greatest and broadest detail on the performance of the economy. Real GDP is a quarterly figure and is released by the Bureau of Economic Analysis.

## Mathematics:

*Exponents:* Recall that for any *positive integer* $n$ , $a^n = \underbrace{a \cdot a \cdot a \cdots a}_{ntimes}$ .

Thus, $a^4 = a \cdot a \cdot a \cdot a$ , and $2^3 = 2 \cdot 2 \cdot 2 = 8$ .

For any real number , $a$ , we define $a^0 = 1$ . Thus, $5^0 = 1$ , $(-9)^0 = 1$ , $(\frac{1}{2})^0 = 1$ , and so on.

To handle *negative exponents* , we define: $a^{-n} = \frac{1}{a^n}$ .

Therefore, $a^{-6} = \frac{1}{a^6}$ , $3^{-2} = \frac{1}{3^2} = \frac{1}{9}$ , $(\frac{2}{3})^{-3} = \frac{1}{(\frac{2}{3})^3} = \frac{1}{\frac{8}{27}} = \frac{27}{8}$ .

Let us now discuss *fractional exponents.* If $a > 0$ , and $n$ is any positive integer, we define $a^{\frac{1}{n}} = \sqrt[n]{a}$ .

Thus, $8^{\frac{1}{3}} = \sqrt[3]{8} = 2$ , and $25^{\frac{1}{2}} = \sqrt{25} = 5$ .

Moreover, if $m$ is any positive integer, we define $a^{\frac{m}{n}} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$ .

Thus, $27^{\frac{2}{3}} = (\sqrt[3]{27})^2 = 3^2 = 9$ , and $25^{-\frac{3}{2}} = \frac{1}{25^{\frac{3}{2}}} = \frac{1}{(\sqrt{25})^3} = \frac{1}{5^3} = \frac{1}{125}$ .

$Order of\ Operations$ :

Let us review the correct **order of operations** that must be used in simplifying an arithmetic expression:

1. First perform all *exponentiation*, proceeding from left to right.
2. Next, perform all *multiplicationsand divisions*, proceeding from left to right.
3. Next, perform all *additions and subtractions* , proceeding from left to right.
4. Simplify expressions in parentheses first, using rules 1. and 2. In the case of more than one set of parentheses, work from the innermost set outwards.

For example, $(\frac{2}{5})^4 - 1 = (.4)^4 - 1 = .0256 - 1 = -.9744$ .

Here, we evaluate what was in the parentheses first , $(\frac{2}{5}) = .4$ Then we exponentiate, $(.4)^4 = .0256.$ Finally, we subtract , $.0256 - 1 = -.9744$ .

When entering an expression on a calculator or computer, you must be careful to include all necessary parentheses. If you fail to do so, the calculator or computer, which follows the correct order of operations, will perform a calculation different from the one that you intended.

For example,

the formula $(\frac{A}{B})^{\frac{1}{5}} - 1$ , must be entered as $(A \div B)\hat{}(1 \div 5) - 1$ .

Thus, to have your calculator evaluate the expression $(\frac{64}{2})^{\frac{1}{5}} - 1$ , you must enter

$(64 \div 2)\ \hat{}\ (1 \div 5) - 1$ and the calculator will do the computation as follows:

$(64 \div 2)\ \hat{}\ (1 \div 5) - 1 = 32^{\frac{1}{5}} - 1 = \sqrt[5]{32} - 1 = 2 - 1 = 1$ .

If you had incorrectly entered $(64 \div 32)\ \hat{}\ 1 \div 5$ , the calculator would have computed $\frac{(\frac{64}{32})^1}{5} = .4$ , because the exponentiation is done before the division.

**Exponential Growth - Compound Interest**
**(See also MIS problem set #1: ABC Company, math section)**

If a quantity Q, grows at a rate of r% a year, then after t years, the quantity has grown to a value Q(t) given by the formula:

$$Q(t) = Q*(1+r\%)^t$$

This is called exponential growth. Money that is compounded at a fixed interest rate grows exponentially.

There is no reason to restrict the compounding time period to a year. A quantity can grow at a fixed percent a quarter (3 month period), or a month. Credit card debt, for example, is grows at a monthly rate.

Most of the time, however, the interest rate is quoted as an annual rate, even though it is understood that the compounding will occur more often. The general understanding is as follows:

Suppose an annual rate of r% is known to compound n times a year. Then, after $t$ year$s$, a quantity Q will grow to:

$$\mathbf{Q(t) = Q * (1 + \frac{r\%}{n})^{n*t}}$$

Example :
Suppose you have $1,000 in an account which pays 5% compounded monthly. What is your balance in two years.

$$Q(2) = 1000 * (1 + \frac{5\%}{12})^{12*2} = 1000 * (1 + .004167) = 1,104.94 \text{ dollars.}$$

Conversely, given the rate per period, we can compute the annual rate. The formula is:

$$\mathbf{r\% = (1 + p\%)^n - 1}$$

where there are n periods per year, and the quantity increases by p% each period.

Example:
Suppose the economy increases at a rate of $\frac{1}{2}$% a quarter. What is the the equivalent annual rate?

$$\text{annual rate} = (1 + \tfrac{1}{2}\%)^4 = ( 1 + .005)^4 = 1.0215 - 1 = 2.15\% \text{ a year.}$$

We can also compute the annual growth rate if we know the amount per period by which the amount increased. The formula is:

$$\textbf{annual rate} = \left(\frac{\textbf{Amount at end of period}}{\textbf{Amount at beginning of period}}\right)^{\mathbf{n}} \mathbf{- 1}$$

where n is the number of periods in the year.

Example: The previous quarter GDP is 6502.3, and the current quater GDP is 6580.8. What is the equivalent annual growth rate?

$$\text{Annual growth rate} = (\frac{6580.8}{6502.3})^4 - 1 = (1.01207)^4 - 1 = 1.049 - 1 = 4.9\%$$

**Excel**

*Entering formulas into a spreadsheet.*

To enter a formula into a spreadsheet, you substitute cell addresses for the variables. These cell addresses contain the values of the variables.

**Example**:

Entering a formula such as (A/B)^(1/5) requires that values for A and B are chosen, and that they are in some cells. The cell address for the value of A is used instead of A, and the cell address containing the value of B is used instead of B.

Suppose A = 10 and B = 8, find (A/B)^(1/5). The spreadsheet with formulas showing looks as follows:

| | A | B |
|---|---|---|
| 1 | A | 10 |
| 2 | B | 8 |
| 3 | | |
| 4 | | =(B1/B2)^(1/5) |

After the numbers are evaluated, we get the following spreadsheet.

| | A | B |
|---|---|---|
| 1 | A | 10 |
| 2 | B | 8 |
| 3 | | |
| 4 | | 1.04564 |

**Displaying the Formulas**

To see the formulas entered onto the spreadsheet, press ctrl-~ (hold down the ctrl key and simultaneously press the tilde (~)). To go back to seeing the numbers, press ctrl-~ again.

**Formatting as Percent**

To change the format of a number to percent, highlight the numbers you want to format, then click the % Icon, or from the menu bar choose "Format" "Cells" "Number" and "Percentage".

**Changing the Number of Decimals Displayed**

To increase or decrease the number of decimal position, highlight the numbers, then click on the "Increase Decimal" or "Decrease Decimal" icon.

**Copying Cells**

Highlight the range of cells you wish to copy, then from the menu bar choose "Edit" "Copy", or click the Copy Icon.  Highlight a range of cells in which to put the copy.  From the menu bar, choose "Edit" "Paste", or else click the Paste Icon.  If a formula was being copied, its cell addresses will change unless they were entered as an absolute address (i.e., $ in front of the letter and number).

You may also copy by dragging.  Highlight the cell whose contents are to be copied.  Move the mouse pointer to the lower right corner of the cell until the + sign appears.  Hold down the left mouse button.  Drag the cell across the cells where you want the copies.

**Business Application**

GDP is a measure of production within the national income and product accounts. There are three alternative ways of deriving GDP: sum of expenditures, sum of incomes, and sum of value added. This problem focuses on the expenditure component since this method is most closely followed by the markets. The major expenditure components are personal consumption (C), gross private domestic investment (I), government purchases (G), and net exports (X-M); they form the familiar identity of:

(1) $GDP = C + I + G + (X-M)$

**The objective of this problem is to develop a spreadsheet model to calculate and analyze GDP using the above identity.**

## Table 1
## GDP Data Worksheet
## (Billions of Chained 1992$)

| | **C** | **I** | **E** | **M** | **G** | **G** |
|---|---|---|---|---|---|---|
| **Period** | **Consumption** | **Investment** | **Exports** | **Imports** | **Federal Government** | **State/Local Government** |
| Annual: | | | | | | |
| 1986 | 3708.7 | 816.4 | 362.2 | 526.1 | 518.4 | 616.9 |
| 1987 | 3822.3 | 826.2 | 402 | 558.2 | 534.4 | 631.8 |
| 1988 | 3972.7 | 830.1 | 465.8 | 580.2 | 524.6 | 656.6 |
| 1989 | 4064.6 | 865.3 | 520.2 | 603 | 531.5 | 682.6 |
| 1990 | 4132.2 | 816.2 | 564.4 | 626.3 | 541.9 | 708.6 |
| 1991 | 4105.8 | 738.1 | 599.9 | 622.2 | 539.4 | 718.7 |
| 1992 | 4219.8 | 790.8 | 639.4 | 669 | 528 | 735.8 |
| 1993 | 4339.7 | 855.4 | 660.6 | 735 | 508.7 | 751.8 |
| 1994 | 4471.1 | 979.9 | 715.1 | 823.3 | 489.7 | 770.5 |
| 1995 | 4578.5 | 1011.5 | 774.8 | 888.9 | 472.7 | 788.6 |
| Quarterly: | | | | | | |
| 1994:1 | 4418.8 | 932.3 | 680.4 | 781.7 | 489.8 | 762.7 |
| 1994:2 | 4457.7 | 985.2 | 704.3 | 816.5 | 483.3 | 766.8 |
| 1994:3 | 4485.8 | 994.7 | 724.8 | 838.1 | 496.6 | 774.7 |
| 1994:4 | 4522.3 | 1007.3 | 751 | 856.8 | 489.1 | 777.7 |
| 1995:1 | 4530.9 | 1024.8 | 755.8 | 874.9 | 481.3 | 782.2 |
| 1995:2 | 4568.8 | 997.8 | 764.3 | 891.2 | 479.9 | 786.3 |
| 1995:3 | 4600.4 | 1015.2 | 779.1 | 893.4 | 472.7 | 791.5 |
| 1995:4 | 4614.1 | 1008.2 | 799.8 | 896.4 | 456.8 | 794.4 |
| 1996:1 | 4655 | 1015.9 | 803.8 | 918.4 | 463.3 | 792.6 |

**The problem contains five parts**:

1. Develop a spreadsheet model to compute GDP using historic expenditure data. Both annual and quarterly data are used.

2. Calculate annual growth rates and quarterly growth rates at an annual rate for GDP. From these calculations, an analysis of how quickly or slowly the economy is growing is made.

3. Calculate average annual compound growth rates for GDP and its components for the periods 1986 to 1991 and 1991 to 1995. By comparing the growth rates averaged over these years, it is possible to determine if the trend in economic performance has changed.

4. Calculate the percent distribution of GDP by expenditure components for each year and quarter. By doing this we can determine which sector is most important in determining GDP.

5. Do a what-if analysis by determining the impact on GDP and the growth rates of a change in one or more of the expenditure components. As an example, examine the following scenario: freeze Federal government spending at its 1986 level and keep it there for the remaining years.

**Spreadsheet Construction**

The data used to calculate GDP is provided in Table 1. Enter the data from Table 1 into an Excel spreadsheet. Each expenditure category goes into a separate column as shown in Table 1. Label each row by its year or quarter. Once the data are entered, the following steps are involved in solving the problem:

*Part One*
Calculate GDP as the sum of the expenditure categories subtracting out imports (use equation 1). Compute GDP in a separate column with a **formula adding up the components and then use the copy command.**

*Part Two*
Next, compute the annual and quarterly growth rates for GDP in a separate column. The formula used to calculate the annual growth rate is:

(current year GDP/previous year GDP) -1

The formula is entered into the spreadsheet beginning with an =, with the current and previous year GDP values (computed in step 1) entered as spreadsheet addresses.

Growth rates are computed for each year from 1987 to 1995 and are formatted as percents.

The formula used to compute the quarterly growth rate as an annual rate is:

((current quarter GDP/previous quarter GDP)^4)-1

Growth rates are computed for each quarter from 1994:2 to 1996:1 and are formatted as percents.

Note: There will be no growth rates calculated for the year 1986 and the quarter 1994:1, since there is no previous annual or quarterly data supplied.

**Discussion question: Looking at the results, why did GDP decline in 1991?**

*Part Three*

Compute the average annual compound growth rates for GDP and its components for the two periods of 1986 to 1991 and 1991 to 1995. Enter the formulas for the growth rates into two rows in the spreadsheet. The following formulas are used to compute the average compound growth rates:

Period 1986-91 (5 years)
((GDP for 1991/ GDP for 1986)^(1/5)) - 1.0

Period 1991-95 (4 years)
((GDP for 1990/ GDP for 1986)^(1/4)) - 1.0

Again, the GDP data is entered as spreadsheet addresses. Repeat these calculation for the components of GDP.

**Discussion question: <u>How do the periods differ? In light of the results is a comparison between the two periods valid?</u>**

*Part Four*

The percent distribution is calculated by taking each GDP category for a given year and dividing it by GDP for that year. This analysis determines how much each of the sectors contribute to GDP and what changes have occurred over time. Again, use the copy command to make the task easier.

**Discussion question: <u>Has there been a change in the distribution over time?</u>**

*Part Five*

The power of a spreadsheet model is readily demonstrated by changing one of the inputs into the model (what-if analysis). In this case, keep Federal Government spending constant at its 1986 level and examine the impact of this scenario on the level of GDP and the growth rates.

**Discussion question: <u>How do the results compare with those found earlier?</u>**

**Answers**

*Parts One and Two*
The values for GDP and the growth rates computed in the spreadsheet are shown in Table 2.

## Table 2
## Results of GDP Calculations
## (Billions of Chained 1992$)

| Period | GDP | Growth Rate |
|---|---|---|
| Annual: | | |
| 1986 | 5496.5 | |
| 1987 | 5658.5 | 2.9% |
| 1988 | 5869.6 | 3.7% |
| 1989 | 6061.2 | 3.3% |
| 1990 | 6137.0 | 1.3% |
| 1991 | 6079.7 | -0.9% |
| 1992 | 6244.8 | 2.7% |
| 1993 | 6381.2 | 2.2% |
| 1994 | 6603.0 | 3.5% |
| 1995 | 6737.2 | 2.0% |
| Quarterly: | | |
| 1994:1 | 6502.3 | |
| 1994:2 | 6580.8 | 4.9% |
| 1994:3 | 6638.5 | 3.6% |
| 1994:4 | 6690.5 | 3.2% |
| 1995:1 | 6700.1 | 0.6% |
| 1995:2 | 6705.9 | 0.3% |
| 1995:3 | 6765.5 | 3.6% |
| 1995:4 | 6776.9 | 0.7% |
| 1996:1 | 6812.2 | 2.1% |

*Part Three*
Table 3 shows the results of the growth rate calculations for the two periods: 1986-91 and 1991-95.

## Table 3
## Average Compound Growth Rates in GDP and the Components of GDP For the Periods: 1986-91 and 1991-95

| Period | Consumption | Investment | Exports | Imports | Federal Government | State/Local Government | GDP |
|---|---|---|---|---|---|---|---|
| 1986-91 | 2.1% | -2.0% | 10.6% | 3.4% | 0.8% | 3.1% | 2.0% |
| 1991-95 | 2.8% | 8.2% | 6.6% | 9.3% | -3.2% | 2.3% | 2.6% |

*Part Four*
Table 4 shows the distribution of GDP by sector.

## Table 4
## Percent Distribution of GDP by Sector

| Period | Consumption | Investment | Exports | Imports | Federal Government | State/Local Government |
|---|---|---|---|---|---|---|
| Annual: | | | | | | |
| 1986 | 67.5% | 14.9% | 6.6% | 9.6% | 9.4% | 11.2% |
| 1987 | 67.5% | 14.6% | 7.1% | 9.9% | 9.4% | 11.2% |
| 1988 | 67.7% | 14.1% | 7.9% | 9.9% | 8.9% | 11.2% |
| 1989 | 67.1% | 14.3% | 8.6% | 9.9% | 8.8% | 11.3% |
| 1990 | 67.3% | 13.3% | 9.2% | 10.2% | 8.8% | 11.5% |
| 1991 | 67.5% | 12.1% | 9.9% | 10.2% | 8.9% | 11.8% |
| 1992 | 67.6% | 12.7% | 10.2% | 10.7% | 8.5% | 11.8% |
| 1993 | 68.0% | 13.4% | 10.4% | 11.5% | 8.0% | 11.8% |
| 1994 | 67.7% | 14.8% | 10.8% | 12.5% | 7.4% | 11.7% |
| 1995 | 68.0% | 15.0% | 11.5% | 13.2% | 7.0% | 11.7% |
| Quarterly: | | | | | | |
| 1994:1 | 68.0% | 14.3% | 10.5% | 12.0% | 7.5% | 11.7% |
| 1994:2 | 67.7% | 15.0% | 10.7% | 12.4% | 7.3% | 11.7% |
| 1994:3 | 67.6% | 15.0% | 10.9% | 12.6% | 7.5% | 11.7% |
| 1994:4 | 67.6% | 15.1% | 11.2% | 12.8% | 7.3% | 11.6% |
| 1995:1 | 67.6% | 15.3% | 11.3% | 13.1% | 7.2% | 11.7% |
| 1995:2 | 68.1% | 14.9% | 11.4% | 13.3% | 7.2% | 11.7% |
| 1995:3 | 68.0% | 15.0% | 11.5% | 13.2% | 7.0% | 11.7% |
| 1995:4 | 68.1% | 14.9% | 11.8% | 13.2% | 6.7% | 11.7% |
| 1996:1 | 68.3% | 14.9% | 11.8% | 13.5% | 6.8% | 11.6% |

*Part 5*
Finally, Table 5 provides the results of the GDP calculation under the scenario that Federal Government spending remains constant at its 1986 level.

## Table 5
## GDP Under the Assumption that Federal Spending Constant At Its 1986 Level

| Period | Consumption | Investment | Exports | Imports | Federal Government | State/Local Government | GDP | Growth Rate |
|---|---|---|---|---|---|---|---|---|
| Annual: | | | | | | | | |
| 1986 | 3708.7 | 816.4 | 362.2 | 526.1 | 518.4 | 616.9 | 5496.5 | |
| 1987 | 3822.3 | 826.2 | 402 | 558.2 | 518.4 | 631.8 | 5642.5 | 2.7% |
| 1988 | 3972.7 | 830.1 | 465.8 | 580.2 | 518.4 | 656.6 | 5863.4 | 3.9% |
| 1989 | 4064.6 | 865.3 | 520.2 | 603 | 518.4 | 682.6 | 6048.1 | 3.2% |
| 1990 | 4132.2 | 816.2 | 564.4 | 626.3 | 518.4 | 708.6 | 6113.5 | 1.1% |
| 1991 | 4105.8 | 738.1 | 599.9 | 622.2 | 518.4 | 718.7 | 6058.7 | -0.9% |
| 1992 | 4219.8 | 790.8 | 639.4 | 669 | 518.4 | 735.8 | 6235.2 | 2.9% |
| 1993 | 4339.7 | 855.4 | 660.6 | 735 | 518.4 | 751.8 | 6390.9 | 2.5% |
| 1994 | 4471.1 | 979.9 | 715.1 | 823.3 | 518.4 | 770.5 | 6631.7 | 3.8% |
| 1995 | 4578.5 | 1011.5 | 774.8 | 888.9 | 518.4 | 788.6 | 6782.9 | 2.3% |
| Quarterly: | | | | | | | | |
| 1994:1 | 4418.8 | 932.3 | 680.4 | 781.7 | 518.4 | 762.7 | 6530.9 | |
| 1994:2 | 4457.7 | 985.2 | 704.3 | 816.5 | 518.4 | 766.8 | 6615.9 | 5.3% |
| 1994:3 | 4485.8 | 994.7 | 724.8 | 838.1 | 518.4 | 774.7 | 6660.3 | 2.7% |
| 1994:4 | 4522.3 | 1007.3 | 751 | 856.8 | 518.4 | 777.7 | 6719.9 | 3.6% |
| 1995:1 | 4530.9 | 1024.8 | 755.8 | 874.9 | 518.4 | 782.2 | 6737.2 | 1.0% |
| 1995:2 | 4568.8 | 997.8 | 764.3 | 891.2 | 518.4 | 786.3 | 6744.4 | 0.4% |
| 1995:3 | 4600.4 | 1015.2 | 779.1 | 893.4 | 518.4 | 791.5 | 6811.2 | 4.0% |
| 1995:4 | 4614.1 | 1008.2 | 799.8 | 896.4 | 518.4 | 794.4 | 6838.5 | 1.6% |
| 1996:1 | 4655 | 1015.9 | 803.8 | 918.4 | 518.4 | 792.6 | 6867.3 | 1.7% |

**Additional Problems**

1. Using the data in Table 1, calculate the average growth rate for GDP and its components for the years 1986-89 and 1992-95?

2. Compute GDP for the years 1986-1995 and for the quarters 1994:q1 to 1996:q1 if exports are equal to imports for the entire period?

3. What happens to GDP, over the period, if exports exceed imports by $25 billion?

**Research Problem:** Use the Internet to extend Table 1 to a more current year. Analyze the trend in GDP over the past few years.