TIME VALUE OF MONEY PROBLEM #4: PRESENT VALUE OF AN ANNUITY

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Introduction

In this assignment we will discuss how to calculate the Present Value of a series of deposits, known as an annuity.

An annuity is a series of payments which are equal and made for a given number of periods. These payments can be made at the end of the period, (an ordinary annuity), or at the beginning of every period, (an annuity due).

If we invest $100 a month for 60 months at the end of each month, what will the value of this amount be in today's dollars at the end of the 60 months? To sove this problem, we need to study the Present Value of an annuity which is the focus of this section.

Mathematics:

Mathematically, the Present Value of an ordinary annuity is computed as follows:

$$\text{PVA} = \frac{\text{Annual Payment} * [\,1 - (1+i)^{-\text{N}}\,]}{i} \qquad [4]$$

where: PVA = Present Value of ordinary annuity
i = Interest rate
N = Number of payments

Example 1 :

If you invest $1,000 at the end of every year for five years and receive a return of 7%, how much is this worth in today's dollars?

$$\text{PVA} = \frac{\text{Annual Payment} * [\,1 - (1+i)^{-\text{N}}\,]}{i}$$

$$\text{PVA} = \frac{1000 * [\ 1 - (1+.07)^{-5}]}{.07} = 4100.20$$

Thus the Present Value of the Annuity is $4,100.20 .

Let us consider what happens if we increase the value of N. Since we are investing $1000 at the end of each year for a *longe*r period of time, we expect that the Present Value of the annuity will be larger.

Example 2 :

Let the Annual Payment = $1,000 , and $i = 7\%$ as in Example 1 , but let us increase N to six years. Then

$$\text{PVA} = \frac{1000 * [\ 1 - (1+.07)^{-6}]}{.07} = 4766.54$$

This example suggests that increasing the value of N results in a *larger* value for PVA, as we expected.

Mathematically we can see that increasing the value of N results in a *smaller* value for the expression $(1+i)^{-\text{N}} = \frac{1}{(1+i)^{\text{N}}}$.

But subtracting this smaller value from 1 will result in a *larger* value in the numerator of formula [4] yielding a larger value for PVA .

Let us now examine what happens if we increase the value of i .

Example 3 :

Let the Annual Payment = $1,000 and N = 5 , as in Example 1 , but let us increase i to 8% . Then

$$\text{PVA} = \frac{1000 * [\,1 - (1 + .08)^{-5}]}{.08} = 3992.71$$

Thus we see that increasing the value of i will cause a *decrease* in thevalue of PVA .

If the payments are made at the beginning of the year (Annuity Due), then the formula for the Present Value of the Annuity Due is found by multiplying the right side of formula [4] by $1 + i$. Thus

$$\text{PVAnnuity Due} = \frac{\text{Annual Payment} * (1 + i) * [\,1 - (1 + i)^{-\text{N}}\,]}{i}$$

Example 4:

If you invest $1,000 at the beginning of every year for five years and receive a return of 7% , how much is this worth in today's dollars?

$$\text{PVAnnuity Due} = \frac{1000 * (1 + .07) * [\,1 - (1 + .07)^{-5}]}{.07} = 4100.20 * 1.07 = 4387.21$$

Using the TI-83:

Compute the Present Value of a stream of payments, in this case $1,000, made at the end of each year for five years, if the interest rate is 7%. What if the payments were at the beginning of each year?

Press [2nd] [FINANCE] [1] [ENTER] to display the TVM solver.

We enter the following:

N = 5
I% = 7
PV = 0.00 This is the value we are solving for.
PMT = 1000
FV = 0.00
P/Y = 1
C/Y = 1
PMT: **END** BEGIN As payments are made at the end of the period.

Next, place the cursor at the variable you are looking to solve. In this case it is the PV (Present Value).

Press [ALPHA] [Solve].

The answer is computed and stored for the appropriate TVM variable. In this case, -4100.197436 will be displayed on the PV amount with an indicated square on the left column, designating the solution variable as follows:

PV = -4100.197436

Thus, $4,100.20 represents the present value of a $1,000 five (5) year ordinary annuity earning a 7% rate of return.

If the payments were made at the beginning of each year (annuity due), the variables entered above would be the same except that we PMT: END BEGIN. After inputting the data we get the solution:

PV = -4387.21

The present value of the payments is $4,387.21.

Business Application:

You have a student loan in the amount of $20,000. The payments will begin in one month over a ten year period at an interest rate of 8% per year, compounded monthly. What will the monthly payments be over this period?

Solution:

This is a Present Value Annuity problem and we will solve for payments (PMT). We enter the following on the TI - 83.

N = 120 (12 months * 10 years)
I% = 8
PV = 20,000
PMT = 0.00 This is the value we are solving for.
FV = 0.00
P/Y = 12 As there are 12 payments per year.
C/Y = 12
PMT: **END** BEGIN. Payments will begin in one (1) month.

Next, put the cursor on PMT and press [ALPHA] [Solve]

We get PMT = -242.65518

The monthly loan payment on this $20,000 student loan at a charge of 8% interest per year is $242.66.

Additional Problems:

1. If you invest $1,000 a year for twenty (20) years at a return rate of 6% per year, what is the Present Value of the annuity?

2. If you invest $1,000 a year for twenty (20) years at a return rate of 6% per year, what is the Present Value of the annuity if payments are made at the beginning of the year?

3. **Research Problem:** Call your local automobile dealer to get a quote on the sales price of your favorite car. Ask your dealer how much your payments will be per month for sixty (60) months for you to buy the car with no down payment. Calculate the interest rate charged on this loan.