Introduction

This assignment will focus on using the TI - 83 to calculate the price of a Zero Coupon Bond. Knowledge of Time Value of Money Problem #2 is a prerequisite towards proceeding with this problem.

A Zero Coupon Bond is one which does not distribute coupon payments semiannually as a traditional bond does. Rather the interest is accrued and paid once: at maturity. In pricing a zero coupon bond, the market uses semiannually compounded interest rate. As such, in calculating the bond’s price, the interest rate is divided by two and the number of payments is the number of years to maturity multiplied by two.

For example, a five year zero coupon bond with a market rate of 6% will have its price calculated as if you received 3% interest, compounded semiannually, for 10 semiannual periods.

Zero Coupon Bonds are also known as discount bonds as these bonds sell well below their face value. An investor will purchase a Zero Coupon Bond at a discount, wait until maturity and receive its face value. Because the bond is purchased at a discount, the face value, say $1,000 represents a payment of both interest and principal.
Mathematics:

A Zero Coupon Bond is simply a Present Value of a single payment problem. The bond is worth the Present Value of the face value received \( N \) compounding periods in the future, where the interest rate per compounding period is \( i \).

\[
\text{Price of Zero Coupon Bond} = FV \times (1 + i)^{-N} \quad [5]
\]

where 
- \( FV \) = Future Value of the bond
- \( i \) = Interest rate per period
- \( N \) = Number of periods

Example:

How much is a $1,000 five year Zero Coupon Bond worth how much today given a 6% market interest rate?

\[
\text{Price of Zero Coupon Bond} = FV \times (1 + i)^{-N}
\]

Remember that you must calculate its worth as if you received 3% interest for 10 semi annual periods. Therefore,

\[
\text{Price of Zero Coupon Bond} = 1000 \times (1 + .06)^{-5} = 744.09
\]

Thus the bond is worth $744.09 today.

As we saw in problem 2, increasing either the value of \( i \) or \( N \) will result in a decrease in the value of the bond, since

\[
(1 + i)^{-N} = \frac{1}{(1+i)^N}
\]

Thus, increasing either \( i \) or \( N \) increases the denominator of the fraction and makes the value of the fraction smaller.
Using the TI-83:

Returning to our previous example, suppose that you want to calculate the price of a $1,000 five year Zero Coupon bond given a 6% market interest rate

Before proceeding with the solution, it is important to note how the TI-83 adjusts for semiannual compounding, when pricing a zero coupon bond. Firstly, the number of periods N equals the number of years to maturity of the bond times two, to reflect the number of semiannual periods. Secondly, I% is the interest rate per year (which is assumed to be compounded semiannually). The fact that interest payments are assumed to be reinvested semiannually to maturity is taken care of by setting the values of P/Y and C/Y to 2.

To price the zero coupon bond, press [2nd] [FINANCE] [TMV Solver]. Then enter the following:

- \( N = 5 \times 2 = 10 \) Number of compounding periods
- \( I\% = 6 \) Market interest rate
- \( PV = 0 \) This is the value we are solving for.
- \( PMT = 0 \) There are no periodic payments
- \( FV = 1000 \) The face value of the bond.
- \( P/Y = 2 \) Semiannual accrual of coupon payment.
- \( C/Y = 2 \) Treats interest rate I% as a semiannually compounded interest rate.

PMT: END BEGIN

Solve for PV by moving cursor next to PV and then pressing [ALPHA] [Solve]

We get, \( PV = -744.0939149 \). This bond is worth $744.09 today.
Business Problem:

Assume that the current market interest rate on a 10 year Zero Coupon Bond is 8%. What effect will changing interest rates have on the price of this bond? What if it were a 30 year zero coupon bond? Assume that the face value is $1,000.

Solution:

First calculate the price of this bond today assuming a $1,000 face value. Using the TI-83, we enter the following:

\[
\begin{align*}
N &= 10 \times 2 \quad \text{Or else, enter 20} \\
I\% &= 8 \\
PV &= 0 \quad \text{This is the value for which we are solving.} \\
PMT &= 0 \\
FV &= 1000 \quad \text{Face value of the bond.} \\
P/Y &= 2 \quad \text{Semiannual accrual of coupon payment.} \\
C/Y &= 2 \quad \text{Treats interest rate I\%, as a semiannually compounded interest rate.} \\
PMT: \text{ END BEGIN} \\
\end{align*}
\]

Solve for PV by putting the cursor next to PV, and then pressing \([ALPHA] \ [SOLVE]\).

We get PV = -456.39. Rounded off to the nearest dollar, the bond is worth $456 today.

By putting in different values of the interest rate, I\%, we obtain Table I which illustrates the effect of changes in market interest rate on the price of a ten year zero coupon bond, with face value of $1,000.

<table>
<thead>
<tr>
<th>Market Rate, I%</th>
<th>Price ($ of Bond)</th>
<th>Change in Price ($)</th>
<th>% Change in Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>11%</td>
<td>343</td>
<td>-113</td>
<td>-25%</td>
</tr>
<tr>
<td>10%</td>
<td>377</td>
<td>-79</td>
<td>-17%</td>
</tr>
<tr>
<td>9%</td>
<td>415</td>
<td>-41</td>
<td>-9%</td>
</tr>
<tr>
<td>8% This bond</td>
<td>456</td>
<td>0</td>
<td>0%</td>
</tr>
<tr>
<td>7%</td>
<td>503</td>
<td>47</td>
<td>10%</td>
</tr>
<tr>
<td>6%</td>
<td>553</td>
<td>97</td>
<td>21%</td>
</tr>
<tr>
<td>5%</td>
<td>610</td>
<td>154</td>
<td>34%</td>
</tr>
</tbody>
</table>
Note that a decrease of each 1% in the market interest rate, increases the price of the bond by roughly 10% or 11%, but an increase of each 1% in market interest rate decreases the price of the bond by roughly 8% or 9%.

Continuing with our previous example, let’s assume that the Zero Coupon Bond has a 30 year maturity date. We keep the market rate at 8%. Calculating the value of the bond using the TI-83, we enter the following:

\[
\begin{align*}
N &= 30 \times 2 = 60 \\
I\% &= 8 \\
PV &= 0 \quad \text{This is the value for which we are solving.} \\
PMT &= 0 \\
FV &= 1000 \\
P/Y &= 2. \\
C/Y &= 2 \\
PMT: \quad \text{END BEGIN}
\end{align*}
\]

Solve for PV by putting the cursor next to PV, and then pressing [ALPHA] [SOLVE].

We get PV = -95.06. The bond is worth $95.06 today.

Table II illustrates the effect of varying market interest rates on the price of a 30 year zero coupon bond.

**TABLE II**
30 Year Zero Coupon Bond

<table>
<thead>
<tr>
<th>Market Rate, I%</th>
<th>Price ($) of Bond</th>
<th>Change in Price ($)</th>
<th>% Change in Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>11%</td>
<td>40.26</td>
<td>-54.80</td>
<td>-57.65%</td>
</tr>
<tr>
<td>10%</td>
<td>53.54</td>
<td>-41.52</td>
<td>-43.68%</td>
</tr>
<tr>
<td>9%</td>
<td>71.29</td>
<td>-23.77</td>
<td>-25.01%</td>
</tr>
<tr>
<td>8%</td>
<td>95.06</td>
<td>0</td>
<td>0%</td>
</tr>
<tr>
<td>7%</td>
<td>126.93</td>
<td>+31.87</td>
<td>+33.52%</td>
</tr>
<tr>
<td>6%</td>
<td>169.73</td>
<td>+74.67</td>
<td>+78.55%</td>
</tr>
<tr>
<td>5%</td>
<td>227.28</td>
<td>+132.22</td>
<td>+139.09%</td>
</tr>
</tbody>
</table>

By comparing Tables I and II, it is evident that the 30 year maturity bond has much greater price fluctuations than the 10 year bond. The greater the number of years to maturity of a bond, the greater the risk becomes. Therefore, greater returns as well as losses can be realized by buying longer term bonds. Additionally, zero coupon bonds are riskier than other types of bonds. This stems from the fact that payments are received only once - at maturity. The fact that there are no intermediate payments make this the riskiest of bonds.
Additional Problem:

1. Find the value of a twenty year zero coupon bond with face of $1,000 if the market interest rate is 5.5%.

2. A seven year Zero Coupon Bond with face value $1,000 is selling at $500.00. Calculate the current market rate of interest.

3. **Research Problem:** Call a broker or use the internet to find out how much twenty year Treasury Zero Coupon Bonds are selling for. Given this information, calculate the annual return of the bond. What will happen to the bond’s price if interest rates fall by 1% one year from now? How about if interest rates rise by 1%?