TIME VALUE OF MONEY PROBLEM #7: MORTGAGE AMORTIZATION

Professor Peter Harris
Mathematics by Sharon Petrushka

Introduction

This problem will focus on calculating mortgage payments. Knowledge of *Time Value of Money Problem #4: Present Value* is a prerequisite towards proceeding with this problem.

A mortgage is simply a loan collateralized by property which is payable over a number of years or on a monthly basis. The lender, usually a bank charges the borrower an interest charge for the privilege of borrowing money.

Mathematics:

A mortgage is simply a stream of payments made by the borrower to the lender. This is an annuity as we discussed in Problem 4 . Mortgages and other loan arrangements, such as an automobile loan, which require monthly payments over a period of time simply involve the use of the Present Value of Annuity concept. The loan amount is the Present Value of the Annuity, the monthly payment is the payment per period, and the cost of borrowing money is the interest rate. Thus,

$$\text{PVA} = \frac{\text{Payment} * [\,1 - (1+i)^{-\text{N}}\,]}{i} \qquad [7]$$

where PVA = Present Value of the loan
i = interest rate
N = number of periods
Payment = Payment per period

In the case of a mortgage, we frequently must find the Payment. We will solve algebraically for Payment in formula [7] . Recall from lesson 2 that we can multiply or divide both sides of an equation by the same non-zero quantity. Thus multiplying both sides of formula [7] by i yields

$$\text{PVA} * i = \frac{\text{Payment} * [\,1 - (1+i)^{-\text{N}}\,]}{i} * i = \text{Payment} * [\,1 - (1+i)^{-\text{N}}\,]$$

and dividing both sides of this expression by $[1 - (1+i)^{-\text{N}}]$ yields

$$\frac{\text{PVA} * i}{[(1 - (1+i)^{-\text{N}}]} = \text{Payment} \qquad [7]$$

Example 1:

If you borrow $200,000 over a five year period at an annual interest rate of 11%, find your yearly payment on the loan?

$$\text{Payment} = \frac{\text{PVA} * i}{[(1 - (1+i)^{-\text{N}}]}$$

$$\text{Payment} = \frac{200{,}000 * .11}{[1 - (1+.11)^{-5}]} = 54,114.06$$

A payment of $54,114.06 will be required yearly for five years to pay this loan off. Note that the payments will be due at the end of each year.

Let us examine what will happen if the interest rate is increased to 15%. Logically, it seems that your payment should also increase.

Example 2:

If you borrow $200,000 over a five year period at an annual interest rate of 15%, find your yearly payment on the loan.

$$\text{Payment} = \frac{200{,}000 * .15}{[1-(1+.15)^{-5}]} = 59{,}663.11$$

We see then that if the interest rate goes up to 15%, your yearly payment will rise to $59,663.11 .

What will happen if we extend the length of the loan to a seven year period?

Example 3:

If you borrow $200,000 over a seven year period at an annual interest rate of 11%, find your yearly payment on the loan.

$$\text{Payment} = \frac{200{,}000 * .11}{[1-(1+.11)^{-7}]} = 42{,}443.05$$

Clearly, extending the length of the loan will result in a decrease in the yearly payment to $42,443.05.

Using the TI-83:

If you took out a twenty-five year mortgage in the amount of $260,000 at an interest rate of 11%, what is your required monthly mortgage payment, assuming that payments are made at the end of the month.

Solution:

Since the payments are required to be made monthly, the number of payments is 25 years * 12 $\frac{\text{payments}}{\text{years}}$ = 300 payments. The interest rate is quoted on an annual basis, but the market convention is that this rate is compounded monthly, and therefore $\frac{p}{y}$ and $\frac{c}{y}$ are both 12.

We enter the following:

N = 300 there are 300 payments
I% = 11
PV = 260000
PMT = 0 This is the value for which we are solving.
FV = 0
P/Y = 12. Monthly payments.
C/Y = 12 Monthly compounded interest rate.
PMT: **END** BEGIN.

Next, place the cursor at the PMT variable, and we then press

[ALPHA] [Solve] .

The answer computed for PMT is -2548.29.

The required monthly mortgage payment for a twenty-five year, $260,000 at 11% interest is $2,548.29.

Business Problem:

You recently purchased a car for $4,500 and are going to finance the car by obtaining a car loan from the bank. The terms of the loan are $4,500 payable at the end of each month for 4 years (48 months) at an annual interest rate of 14.90%. Find your required monthly payments on the loan?

We enter the following:

N = 48 there are 48 payments.
I% = 14.9
PV = 4500
PMT = 0 This is the value for which we are solving.
FV = 0
P/Y = 12. Monthly payments.
C/Y = 12 Monthly compounded interest rate.
PMT: **END** BEGIN.

Next, put the cursor on the 0 next to PMT, and press

[ALPHA] [SOLVE].

The answer computed for PMT is $125.01. The required car payment over a 48 month period (4 years) at 14.90% annual interest is $125.01.

Additional Problems:

1. You want to borrow $10,000 to buy a car. The loan is at 10% per year and is to be paid over forty eight months. What is the monthly loan payment?

2. A $100,000, ten year mortgage with an 11% interest rate will require monthly payments in what amount?

3. **Research Problem**: If you were to take out a student loan in the amount of $10,000 what will be the monthly payments given prevailing interest rate of loans and the time period of the loan? (Call your Financial Aid Office).