\[ f(2,3) = \sqrt{6}/2-1 = 1/2 \]

\[ f(x) = \frac{x}{x+y+1} \]

\[ D = \{ (x,y) \mid x+y+1 \geq 0 \} \]

Find the domain of \( f \) and \( f(2,3) \)

\[ y = f(x) \]

\[ D = \{(x,y) \mid f \text{ is defined} \} \]

\[ \text{Two variables, two domains} \]

\[ f(x,y) \mid (x,y) \in D \]

14.1 Functions of Several Variables
ex) \( g(x, y) = \sqrt{9 - x^2 - y^2} \)

Find the domain & range of \( g \).

\[ 9 - x^2 - y^2 \geq 0 \implies 0 \leq x^2 + y^2 \leq 9 \]

\[ -9 \leq -x^2 - y^2 \leq 0 \]
\[ \sqrt{0} \leq \sqrt{9 - x^2 - y^2} \leq \sqrt{9} \]

\[ 0 \leq z \leq 3 \]

ex) Sketch the graph of \( f(x, y) = 6 - 3x - 2y \).

\[ z = 6 - 3x - 2y \implies 3x + 2y + z = 6 \quad \text{plane} \]

ex) Sketch the graph of \( g(x, y) = \sqrt{9 - x^2 - y^2} \).

\[ z = \sqrt{9 - x^2 - y^2} \]
\[ x^2 + y^2 + z^2 = 9 \]
Level Curves.

The level curves of a function $f(x,y)$ are the curves with equations $f(x,y) = k$, where $k$ is a constant.

Example:

- Common example of level curves occurs in topographic maps of mountainous regions. (You will never ascend or descend if you walk along one of the levels.)
ex) Sketch the level curves of the function $f(x,y) = 6 - 3x - 2y$ for the values $k = -6, 0, 6, 12$.

\[-6 = 6 - 3x - 2y \implies 3x + 2y = 12\]

\[0 = 6 - 3x - 2y \implies 3x + 2y = 6\]

\[6 = 6 - 3x - 2y \implies 3x + 2y = 0\]

ex) Sketch the level curves of the function $g(x,y) = \sqrt{9-x-y}$ for $k = 0, 1, 2, 3$.

A contour map of a function

Estimate $f(2,1) \approx 60$.

$f(4,3) \approx 78$. 